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**MATHEMATICAL
INDUCTION**

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EXPONENTIAL AND LOGARITHMIC SERIES & MATHEMATICAL INDUCTION

EXPONENTIAL & LOGARITHMIC SERIES

1. The number e

The sum of the infinite series $1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots + \infty$ is denoted by the number e.

$$\text{i.e., } e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots + \infty$$

$$\therefore e = \sum_{n=0}^{\infty} \frac{1}{n!} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$$

Note :

- (i) The number lies between 2 and 3. Approximate value of $e = 2.718281828$.
- (ii) e is an irrational number. (i.e., $e \notin \mathbb{Q}$)

2. Exponential Series

Expansion of any power x to the number e is the exponential series.

$$\text{i.e., } e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \infty = \sum_{n=0}^{\infty} \frac{x^n}{n!} \quad (\text{where } x \in \mathbb{R})$$

- (i) Exponential theorem :

Let $a > 0$ then for all real value of x,

$$a^x = 1 + x(\log_e a) + \frac{x^2}{2!}(\log_e a)^2 + \frac{x^3}{3!}(\log_e a)^3 + \dots = \sum_{n=0}^{\infty} \frac{(x \log_e a)^n}{n!}$$

- (ii) Some standard deductions from exponential series :

$$(i) \quad e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots + \frac{(-1)^n}{n!} x^n + \dots + \infty \quad \{\text{Replace } x \text{ by } -x\}$$

$$e^{-x} = \sum_{n=0}^{\infty} (-1)^n \cdot \frac{x^n}{n!}$$

$$(ii) \quad e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots + \infty \quad \{\text{Putting } x = 1 \text{ in } (e^x)\}$$

$$e = \sum_{n=0}^{\infty} \frac{1}{n!}$$

$$(iii) \quad e^{-1} = 1 + \frac{1}{1!} - \frac{1}{2!} + \frac{1}{3!} + \dots + \infty \quad \{\text{Putting } x = -1 \text{ in } (e^x)\}$$

$$e^{-1} = \sum_{n=0}^{\infty} (-1)^n \cdot \frac{1}{n!} \quad x = -1$$

$$(iv) \quad \frac{e^x + e^{-x}}{2} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots + \infty$$

$$\frac{e^x + e^{-x}}{2} = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$$

$$(v) \frac{e + e^{-1}}{2} = 1 + \frac{1}{2!} + \frac{1}{4!} + \dots \infty$$

$$\frac{e^1 + e^{-1}}{2} = \sum_{n=0}^{\infty} \frac{1}{(2n)!}$$

3. Logarithmic Series : If ($|x| < 1$), then

$$\log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \infty = \sum_{n=1}^{\infty} (-1)^{n-1} \cdot \frac{x^n}{n} \text{ is called as logarithmic series.}$$

Some standard deductions from logarithmic series :

$$(i) \log(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} + \dots \infty = -\sum_{n=1}^{\infty} \frac{x^n}{n}$$

$$(ii) \log(1+x) - \log(1-x) = \log\left(\frac{1+x}{1-x}\right) = \left(x + \frac{x^3}{3} + \frac{x^5}{5} + \dots\right)$$

$$(iii) \log(1+x) + \log(1-x) = \log(1-x^2) = -2\left(\frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{6} + \dots\right)$$

Note :

(i) Napierian or Natural log can be converted into common by using following relation :

$$\log_{10} N = \log_e N \times 0.43429448$$

$$(ii) \log 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \dots = \frac{1}{2.1} + \frac{1}{3.4} + \frac{1}{5.6} + \dots$$

MATHEMATICAL INDUCTION

Mathematical statement:

Statements involving mathematical relations are known as the mathematical statements. For example: 2 divides 16, $(x+1)$ is a factor of x^2-3x+2 .

The principle of mathematical induction:

(i) First principle of mathematical induction:-

Let $P(x)$ be a statement involving the natural number in such that

(i) $P(1)$ is true i.e. $P(n)$ is true for $n = 1$.

(ii) $P(m+1)$ is true whenever $P(m)$ is true.

then $P(n)$ is true for all natural numbers n .

(ii) Second principle of mathematical induction:

Let $P(n)$ be a statement involving the natural number n such that

(i) $P(1)$ is true

(ii) $P(m+1)$ is true, whenever $P(n)$ is true $\forall n$, where $1 \leq n \leq m$.

then $P(n)$ is true for all natural numbers.